NUMERICAL SOLUTIONS OF EQUATIONS DESCRIBING NONISOTHERMAL FLOWS OF A REAL GAS IN TUBES

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A description is given for the numerical method of calculating nonsteady, nonisothermal flows of a real gas in tubes and for the solution on a computer of certain specific problems associated with the operation of gas mains.

The development of methods to solve equations of nonsteady nonisothermal gas motion in tubes is of no mean practical interest in view of the calculations associated with start-up, emergency, and similar

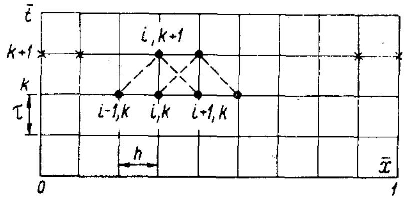

Fig. 1. Calculation scheme.

specific regimes of drilling operations, as well as industrial and major gas-main conduits, particularly in the regions of the Far North.

Since the flow of the gas under actual conditions is nonisothermal, the solutions of the problems must be based on consideration of the general equations of gasdynamics. It is impossible to derive analytical solutions for these equations and it therefore becomes

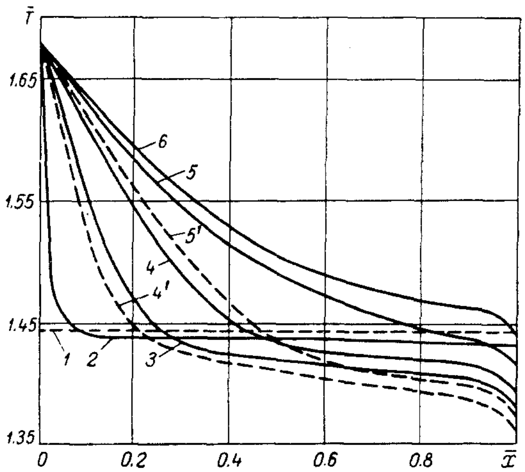

Fig. 2. Change in gas temperature along a pipeline at different instants of starting period 1) $\bar{t} = 0$; 2) 0.03; 3) 0.78; 4) 10.5; 5) 30.5; 6) 40.5 (ideal gas); 4') 10.5; 5') 30.5 (real gas).

necessary to resort to numerical methods involving the use of contemporary computers. Effective numerical methods have been developed in recent years to solve the general equations of gasdynamics. In particular, these include the finite-difference methods based on the use of the so-called implicit difference schemes (for example, schemes of the "predictor-corrector" type and their various modifications [1,2]). These schemes exhibit elevated stability and ensure a rather high order of accuracy for the approximation of differential equations by difference equations. Here the general equations of nonsteady nonisothermal motion of a real gas are reduced to a system of quasilinear differential equations of the parabolic type. For the numerical solution of the indicated equations we will subsequently use explicit finite-difference schemes which have proved to be rather effective in calculating

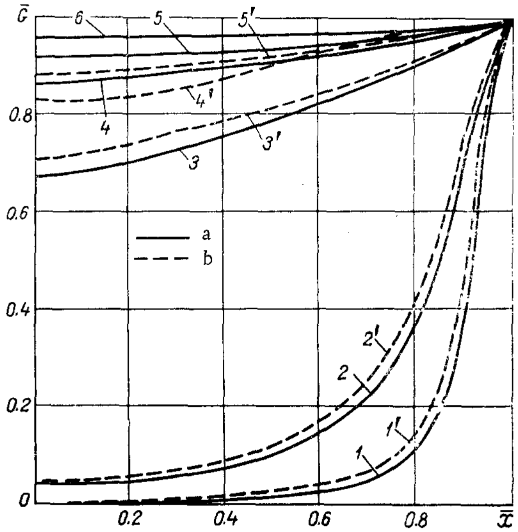

Fig. 3. Change in gas flow rate along a pipeline at different instants of starting period 1) $\bar{t} = 0.012$; 2) 0.112; 3) 8.012; 4) 28.01; 5) 40.4; 6) 60.5.

a number of specific problems in hydrogasdynamics, particularly in calculating the nonsteady isothermal motions in gas mains [3].

1. In deriving the system of equations describing the one-dimensional nonsteady nonisothermal flow of a real gas in long tubes, as is usual [4], we neglect the changes in the dynamic head and in geometric height. Moreover, we do not take into consideration the transfer of heat along the axis of the tube as a result of heat conduction. The heat-transfer law is assumed in Newton's form, with the ambient temperature held to be a known function of the coordinates.

Having used the known thermodynamic relationships and equations of motion for a compressible gas in the Charnyi form [4], we obtain

$$\Delta \frac{\partial P}{\partial t} = \frac{1}{4ab}\frac{1}{G}\frac{\partial^2 P^2}{\partial x^2} + \frac{1}{2a} G \frac{\partial}{\partial x}(z_0 T) - \left(\frac{1}{T} + \frac{1}{z_0}\frac{\partial z_0}{\partial T}\right) \times \left[\frac{1}{a} G z_0 T \frac{\partial T}{\partial x} + c\,\frac{G^3 T^4 z_0^2}{P^2}\frac{\partial z_0}{\partial T} - h^* z_0 T (T_0 - T)\right],$$

$$\Delta \frac{\partial T}{\partial t} = m\left[\frac{1}{4ab}\frac{1}{G}\frac{\partial^2 P^2}{\partial x^2} + \frac{1}{2a} G \frac{\partial}{\partial x}(z_0 T)\right] \times \left(z_0 + \frac{\partial z_0}{\partial T}\right)\frac{T}{P} + \left(\frac{P}{z_0}\frac{\partial z_0}{\partial P} - 1\right) \times \left[\frac{1}{a}\frac{G z_0 T}{P}\frac{\partial T}{\partial x} + c\,\frac{G^3 T^4 z_0^2}{P^3}\frac{\partial z_0}{\partial T} - h^* \frac{z_0 T}{P}(T_0 - T)\right], \quad \frac{\partial P^2}{\partial x} = -2bG^2 z_0 T,$$

$$\Delta = 1 - \frac{P}{z_0}\frac{\partial z_0}{\partial P} - mT\left(z_0 + T\frac{\partial z_0}{\partial T}\right) \times \left(\frac{1}{T} + \frac{1}{z_0}\frac{\partial z_0}{\partial T}\right); \quad a = \frac{f}{R}; \quad b = \frac{\lambda R}{2gDf^2};$$

$$c = \frac{\lambda A R^3}{2gDf^3 c_p}; \quad m = \frac{AR}{c_p}; \quad h^* = \frac{K \pi D R}{c_p f}. \qquad (1)$$

Equation (1) contains the functions $z_0 = z_0(P, T)$ and $c_p = c_p(P, T)$, which may be given in tables of experimental data or in analytical form.

The method for the numerical solution of the equations of nonsteady nonisothermal motion of a real gas, considered below, makes it possible to employ the equation of state in any form.

The temperature T_0 of the external medium is contained in system (1). In view of the considerable lag in the process of heat propagation through soil, it may be regarded as quasi-steady [5], i.e., we can assume T_0 = const in this time interval.

For the practical purposes of interest here, as well as in the light of the unavoidable and considerable errors in the specification of thermophysical soil constants which, moreover, vary along the length of the conduit, we will assume that the heat-transfer coefficient K = const.

The quantity K can be determined from studies under actual conditions, if the parameters of the operating gas main are known.

The calculation of the nonsteady nonisothermal motion of a real gas thus reduces to finding the solution for system (1)

$$P = P(x, t), \quad T = T(x, t), \quad G = G(x, t)$$

in the region $D^*(\alpha \le x \le \beta,\ t \ge 0)$, satisfying the initial conditions

$$P(x, 0) = f(x), \quad T(x, 0) = \varphi(x), \quad G(x, 0) = \psi(x) \quad (2)$$

and the boundary conditions which, in the general case, are given by functional relationships of the form

$$\Phi_1(P, T, G, t)\,|_{x=\alpha} = 0, \quad \Phi_2(P, T, G, t)\,|_{x=\beta} = 0. \quad (3)$$

It is assumed that to the instant of time $t = 0$ the motion of the gas is steady ($P = P(x)$, $T = T(x)$; $G = G_0$ for $t \le 0$, $\alpha \le x \le \beta$). This solution can be obtained from the system of equations which becomes (1), if the time derivatives with respect to the sought

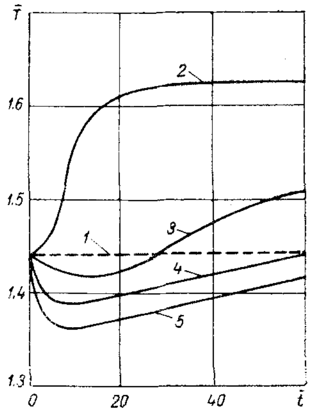

Fig. 4. Temperature history at different points of a pipeline 1) $\bar{T}(\bar{x}, 0)$; 2) $\bar{x} = 0.1$; 3) 0.5; 4) 0.9; 5) 1.0.

functions are assumed to be equal to zero. For convenience of calculation, system (1) is brought to dimensionless form; the scales employed here are the critical pressures and temperatures ($\bar{P} = P/P_c$, $\bar{T} = T/T_c$), the tube length ($\bar{x} = x/L$), the mass flow rate at $t = 0$, ($\bar{G} = G/G_0$) and $t_0 = L/c_0$ ($\bar{t} = t/t_0$, c_0 is the speed of sound in the gas).

2. This boundary problem was solved numerically. Having used the corresponding approximation of the derivatives in (1), we obtained the explicit four-point scheme of the grid method for the system of differential equations (Fig. 1). (It should be noted that when $z_0 = 1$, the general difference scheme changes into the corresponding scheme for the equations describing the flow of an ideal gas). In the construction of the difference scheme we approximated the differential equation by means of difference equations with errors of order O(h).

The indicated order of error is completely acceptable when the sought functions need not be calculated with very high accuracy (within the specified limits of accuracy for the initial data), which corresponds to the conditions of most engineering calculations. The corresponding approximation of the conditions at the boundaries (for boundary conditions of the form $\bar{P}(0, \bar{t}) = \bar{f}_1(\bar{t})$; $\bar{G}(1, \bar{t}) = \bar{f}_2(\bar{t})$ in the assumption that Eqs. (1) are also valid at the boundaries $x = 0$ and $\bar{x} = 1$) leads to the difference relationships:

$$\bar{P}_{0,\,k+1} = \bar{f}_1(\bar{t}_{k+1}), \qquad \bar{T}_{0,k+1} = \frac{\bar{P}_{0,k+1} - A_0}{B_0},$$

$$\bar{G}_{0,\,k+1} = \left[\frac{1}{b\,(z_0\bar{T})_{0,\,k+1}}\,\frac{\bar{P}^2_{1,k+1} - \bar{P}^2_{0,k+1}}{h}\right]^{1/2}, \quad (4)$$

where

$$A_0 = A_0(\bar{P}_{0,k}, \bar{T}_{0,k}, \bar{G}_{0,k}, \bar{T}_{1,k});$$

$$B_0 = B_0(\bar{P}_{0,k},\ \bar{T}_{0,k}), \quad k = 0,\ 1,\ 2,\ \ldots;$$

$$\bar{P}^2_{n,k+1} - \bar{P}^2_{n-1,k+1} =$$

$$= -ch\,\Phi_1\left[z_0(\bar{P}_{n,k+1}, \bar{T}_{n,k+1}),\ \bar{T}_{n,k+1}\right];$$

$$\bar{T}_{n,k+1} = \frac{\bar{P}_{n,k+1} - A_1}{B_1}; \quad \bar{G}_{n,\,k+1} = \bar{f}_2(\bar{t}_{k+1}),$$

$$A_1 = A_1(\bar{P}_{n,k}, \bar{T}_{n,k}, \bar{T}_{n-1,k}), \quad B_1 = B_1(\bar{P}_{n,k},\ \bar{T}_{n,k}),$$

$$k = 0,\ 1,\ 2,\ \ldots. \tag{5}$$

The order of magnitude for the error in the approximation of the boundary conditions is O(h).

Determination of $\bar{P}_{n,\,k+1}$ at the boundary nodes on the straight line $\bar{x} = 1$, reduces to finding the positive root of the nonlinear algebraic equation (5) (in the case of an ideal gas, to the finding of the square root; for a real gas which, for example, follows the Berthelot equation of state, to the finding of the complete fourth-degree algebraic equation).

To calculate $\bar{P}_{n,k+1}$ we use the method of iterations, with the iterative process built on the formula

$$\bar{P}^{(n+1)}_{n,k+1} = \varphi(\bar{P}^{(n)}_{n,k+1}) \equiv \left[\bar{P}^2_{n-1,\,k+1} - ch\Phi_1(\bar{P}^{(n)}_{n,k+1})\right]^{1/2}. \tag{6}$$

The iteration process (6) converges if the following condition is satisfied:

$$\frac{ch\Phi_1'(\bar{P})}{\left[\bar{P}^2_{n-1,\,k+1} - ch\Phi_1(\bar{P})\right]^{1/2}} < 2 \quad (0 < \bar{P} < \bar{P}_{n-1,\,k+1}). \tag{7}$$

Analysis of expression (7) shows that the convergence of the iterations is ensured for rather small h and fixed $\bar{P}_{n-1,\,k+1}$, different from zero.

The corresponding algorithms for the solution of certain specific problems such as, for example, the problem of stabilizing the temperatures and pressures of a real gas in an inactive main [6], etc., can be derived from the cited difference scheme.

The presence of the term $\partial^2 P^2/\partial x^2$ in the equations of system (1) from the conditions of stability imposes on steps h and τ a limitation of the form $\tau = O(h^2)$. Practically speaking, for a tentative selection of the time step τ we employed the stability condition $\alpha\tau/h^2 \leq 1/2$, occurring in the linear case [7]. In application to the quasi-linear system (1), this limitation is not too rigid and, as demonstrated by the calculations, the steps τ may be relatively large, commensurate with the time of a real physical process (for example, when $\alpha = 1$, the difference scheme remains stable for several forms of boundary and initial conditions). This circumstance in particular makes feasible the use of the explicit scheme in solving the problems under consideration.

3. To calculate the nonsteady nonisothermal flows of a real gas in long tubes, we compiled a program in accordance with the subject numerical method of solving Eqs. (1). The calculations were carried out on the BESM-2M computer.

Here, for convenience of machine operation and to make possible future derivations of approximate analytical solutions, $z_0(P, T)$ is taken from the Berthelot equation of state [8]. The possibility of using this equation for practical purposes is demonstrated in [9].

As an example we have examined the start-up regime in a segment of a gas main for the following initial data: L = 100 km; D = 0.7 m, G_0 = 100 kg/sec; $P_0 = 55 \cdot 10^4$ kg/m^2; $P_c = 45.8 \cdot 10^4$ kg/m^2; T_c = 190.6° K; λ = 0.012; K = 2.32 W/m^2·deg; the gas is methane. It is assumed that prior to the instant of time $\bar{t} = 0$ the pressure and temperature of the gas were constant ($\bar{P}(0, \bar{x}) = 1.2$; $\bar{T}(0, \bar{x}) = 1.44$). At the instant of time $\bar{t} \geq 0$ a compressor station was connected to the beginning of the gas-main segment, with a constant pressure $\bar{P}(0, \bar{t}) = 1.2$ and a gas temperature $\bar{T}(0, \bar{t}) = 1.67$ maintained at the outlet from the compressor. It is assumed here that the removal of the gas at the end of the gas main (x = 1) remains constant in time, i.e., $\bar{G}(1, \bar{t}) = 1$.

At the beginning of the process ($\bar{t} \geq 0$) (Fig. 2), three zones can be conditionally isolated: 1) the heated zone near the left boundary ($\bar{x} = 0$), where the temperature drops sharply to the steady value; 2) the region with a temperature close to a steady magnitude; 3) the zone with a temperature less than the steady value (i.e., the temperature of the soil). With the passage of time the first and third zones increase, while for $\bar{t} \cong 10$ the second zone disappears. This interval is approximately equal to the time of a five-fold passage of a pressure wave from $\bar{x} = 0$ to $\bar{x} = 1$ and vice versa.

During the initial instants of time (to $\bar{t} \leq 0.01$) virtually half the tube ($0 \leq \bar{x} \leq 0.5$) remains in the unperturbed state ($\bar{G} = 0$; $\bar{P} = \bar{P}_1$) (see Fig. 3). Only after $\bar{t} = 0.1$ does the gas begin to move at the left end ($\bar{G} > 0$).

With the passage of time, the mass flow rate along the length of the tube increases (Fig. 3), whereas the pressure drops, and when $\bar{t} > 60$ with an error on the order of 5% a steady value is attained ($\bar{G} = 1$; $\bar{P} = \bar{P}(\bar{x})$).

The gas temperature at the various points of the gas conduits increases with time and when $\bar{t} > 60$ tends toward the steady state. At the points closest to the end of the tube from which the gas is removed, the temperature of the gas, with the passage of time, drops below the temperature of the soil as a result of the Joule-Thomson effect $\left(\left(\frac{\partial \bar{P}}{\partial \bar{t}}\right)_{\bar{x}=1} < 0\right)$ (Fig. 4). Starting with the instant of time $\bar{t} \simeq 10$, at which the front of the warmer gas reaches the end of the tube, the drop in temperature due to the Joule-Thomson effect will be attenuated as a result of the influx of heat from the moving gas flow, while with a predominance of heat influx the temperature of the gas will rise with the passage of time. However, at the end of the tube ($\bar{x} = 1.0$), the Joule-Thomson effect is not completely offset and the temperature of the gas will not reach $\bar{T}_0$. We see from Fig. 4 that at the tube points $\bar{x} \leq 0.5$ the steady regime is reached when $\bar{t} \leq 30$, while when $\bar{x} > 0.5$ this time is increased by a factor of 2–3. During the time interval in which the pressure wave covers a distance (60–100) L, a conditionally steady thermal regime is established in the gas conduit.

If we know the thermal and hydraulic regimes of the gas main at the various instants of the start-up period, we can forecast the dynamics for the formation of hydrate plugs, zones of condensate precipitation, etc. The time for the calculation of this problem is approximately 5 hr for a magnitude of $h = 0.05$.

With this program we have calculated the distribution of temperatures and pressures in an inactive conduit [6]. To check on the accuracy of the calculation of the sought functions, we carried out calculations with half a step. The subsequent comparison of the corresponding results demonstrated their excellent agreement, indicating the virtual convergence of the approximate difference solution with the exact.

The subject method of numerical solution for the equations of nonsteady nonisothermal motion of a real gas may be used in gas thermodynamic calculations of major gas-conduit systems.

NOTATION

G is the mass flow rate; P is the pressure; T and T_0 are the temperatures of the gas and soil, respectively; λ is the hydraulic resistance coefficient; D and f are the diameter and cross-section of the tube; z_0 and R are the compressibility factor and gas constant, respectively; A is the thermal equivalent of work; c_p is the isobaric heat capacity; K is the heat transfer coefficient from gas to soil; t is the time; x is the coordinate.

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